### UDC 62-843.6

## THE OPTIMAL COMMERCIAL SPEED OF A TRANSPORT VESSEL WITH RESPECT TO OPERATORS' SUBJECTIVE PREFERENCES

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By a ship's and her engine technical data, it is considered economical modes of the ship's power plant equipped with the main engine of 6L42MC operation. A mathematical modeling of the ship's optimal commercial speed has been conducted. The minimum of the operational costs and maximum of the profit for a voyage were chosen as the criteria of the optimization. Allowance was made for specific brake fuel-oil consumption, subjective preferences of operators. Analytical expressions of parameters used for the optimization have been achieved. Dependences are illustrated by graphs.

*Key words: ship propulsion, main engine, mode of operation, optimal commercial speed, subjective preferences, multi-alternative situations, expectation.* 

**Introduction.** At the ship propulsion and power plants department of Kherson State Marine Academy there elaborated a methodic for determination of the economical efficiency of a ship's propulsion and her power plants (SPPP's) operation. Particularly, an approach to such a problem solution is based upon an optimization problem setting which considers elements of profit maximization, and operational costs minimization.

On the other hand there are always some factors of an uncertainty when operating the SPPPs. Thus, this aspect is also influential.

**Urgency of researches.** The problem of monitoring and supporting the proper technical state of marine ships' propulsions and their power plants in multialternative operational situations is a complex and actual one. It is always important to keep the main engines (MEs) and SPPPs' up states and operate them economically (with the minimum costs) and effectively (with the maximum profits).

The subjective preferences of responsible engineers have to be taken into account as well, because their decisions reflect a measure of the uncertainty at the actual choice of a certain operational mode.

The given problem setting in the general view has a connection with the optimization of the technical-economical policy of a transportation company on the conditions of the lack or limit of the resources when choosing an alternative strategy in order to saving finances and at the keeping the proper technical state of the SPPPs.

Analysis of the latest researches and publications. At the work [1, p. 114], it is considered the optimization of the installed on board ship ME. Also there [1, P. 152-156], it is considered the change of the thermal efficiency index – specific brake fuel-oil consumption (SBFOC) for a slow speed engine (SSE) by the nomogram by «MAN Diesel» concern. In the result of the SBFOC decrease determination, for corresponding outputs of the optimized at certain points ME, the graph shown in the figure [1, p. 156, fig. 51] is drawn, which demonstrates the character of the dependence of the SBFOC upon the ME load. However, for

operation process the optimum of this criterion is not sufficient, because it does not take into consideration all other voyage expenditures. The attempt to an approach to evaluation for such a kind of a problem has been taken in the works [2, 3, P. 169-180]. Although, there were no income and profit aspects taking into account, there. That concept has been illustrated in the work [4, p. 185]. However, there were no mathematical ideas and scientific explanations or proofs for the concept. Moreover, the income curve has a linear character which seems at least rather doubtful.

Theoretical concepts for taking into account the subjective preferences have been developed in the monographs dedicated to the theory of subjective analysis [5, 6].

The task setting. Thus, the purpose of this paper is to find a principle for the idea of a ship speed optimization and compile a mathematical model that takes into account the main conceptual notions of reliability, costs, incomes, profits, and subjective preferences.

The main content (material). After plotting the dependence of SBFOC upon the load of the ME by the nomogram for a SSE, it is determined an hourly fuel-oil consumption at a certain load and corresponding rotational speed of the engine crankshaft. Then, it is accomplished the transition with the approximation to the dependences between expenses and the ship's speed. First, it is being considered the two components of the expenses per an hour. Namely, they are such expenses that depend upon the ship's speed and the other ones that do not depend on it. Knowing the distance of the voyage route we get the duration of the voyage. In the simplest problem setting, at the constant ship's speed and unchangeable in time incomes, the minimum of the voyage expenditures will show the optimal commercial speed of the goods transportation, which should be referred to as for the opportunity to be acceptable in the given voyage on conditions of it appropriateness subject to technical possibility and the actual state of the SPPPs.

In the given problem setting the attention is also paid to the consideration of the influence upon the profitableness of the operation of both the income and cost member components of the formula for the profitableness determination, since, the operation of the SPPPs is directed upon accomplishing the transportation work, and for that case the value of transported load and goods can change in the course of time, by this, the amount of incomes obtained from the voyage is also changing.

Then, there is a problem of choice of the optimal commercial speed with respect to the up state reliability measure of the SSE and subjective preferences of it.

**The problem formulation.** The sense of a transportation vessel operation is to obtain profit. The profit is determined as the difference between income and expenses:

$$P_r = I_n - C_o, \tag{1}$$

where  $P_r$  – the profit;  $I_n$  – the income;  $C_o$  – the cost of, correspondingly finance resources obtained from and spent upon a ship operation.

Minimizing expenses for the operation at the given income from it, there can be got the maximum profit:

$$P_{r\max} = \arg\min_{p} C_{o}.$$
 (2)

Analogous, at the given amount of expenditures, in order to obtain the maximum profit, one has to increase the income from the operation:

$$P_{r_{\max}} = \arg\max_{P_r} I_n.$$
(3)

We will take into account the incomes for a voyage, with respect to the natural loss of the goods value, by the well known exponential dependence of decay and multiplication, as the solution of a corresponding differential equation. The condition is the rate of the quality loss of the goods and loads, which is directly proportional to the goods amount on board ship at the every certain moment of time. The law of the change of the value of the loads and goods will be determined dependently upon the time. If at the moment of time t = 0 the quality mass of the goods is  $m_0$ , and the price  $-c_0$ , initial value of the income  $-C_0 = m_0 c_0$ .

The rate of the value decrease will be determined in the next way. Let at the moment t, the value was  $-\$_t$ , at the moment  $t + \Delta t$  – the value  $-\$_t + \Delta\$_t$ . In the time period of  $\Delta t$  the value decreased by the  $\Delta\$_t$ . The ratio of  $\frac{\Delta\$_t}{\Delta t}$  is a mean rate of the decrease of the value. The limit of the ratio at the  $\Delta t \rightarrow 0$ :

$$\lim_{\Delta t \to 0} \frac{\Delta \$_t}{\Delta t} = \frac{d\$_t}{dt}$$
(4)

is the rate of the value of the goods decrease at an instant t.

On conditions:

$$\frac{d\$_{t}}{dt} = -n \cdot \$_{t}, \tag{5}$$

where n – coefficient of proportionality (n > 0). For the given case, the coefficient of n determines the stableness or stability of the loads value. We use the negative sign because at the increase of time the quality of the goods decreases, thus,  $\frac{d\$_{t}}{d} < 0$ .

The equation (5) of the rate of the value of the goods decrease at an instant t is an equation with separable variables. The solution is well known:

$$\frac{d\$_{t}}{\$_{t}} = -n \cdot dt , \ln \$_{t} = -n \cdot t + \ln C , \qquad (6)$$

where C – the constant of integration, from where:

$$\ln \frac{\$_{t}}{C} = -n \cdot t, \ \frac{\$_{t}}{C} = e^{-n \cdot t}, \ \$_{t} = C \cdot e^{-n \cdot t}.$$
(7)

Since at t = 0 the initial value of the income was  $C_0$ , then *C* should satisfy the interrelationship:

$$C_0 = C \cdot e^{-n \cdot 0} = C. \tag{8}$$

Substituting the value of C (8) into the equation of the loads value (7), we will get the searched dependence of the voyage incomes:

$$I_{n} = \$_{0}(t) = \$_{t} = C_{0} \cdot e^{-n \cdot t} = m_{0} \cdot c_{0} \cdot e^{-n \cdot t}.$$
(9)

The coefficient of *n* is determined by the observations in the next way. Let it be in the time interval  $t_0$  the load and goods value decreases by  $\alpha$  % from the previous magnitude. Thus, the interrelationship is accomplished:

$$\left(1 - \frac{\alpha}{100}\right) \cdot C_0 = C_0 \cdot e^{-n \cdot t_0}, \quad -n \cdot t_0 = \ln\left(1 - \frac{\alpha}{100}\right), \quad n = -\frac{1}{t_0} \cdot \ln\left(1 - \frac{\alpha}{100}\right). \tag{10}$$

For the cost part member of the profit formula (1), it is considered a financial estimate of the hourly fuel-oil consumption:

$$\$_{G_t} = f_1(G_t), \$_{g_e} = f_2(g_e), G_t = f_3(v_s), \$_{G_t} = f_1(f_3(v_s)) = f_4(v_s),$$
(11)

where  $G_t$  – the hourly fuel-oil consumption;  $g_e$  – the SBFOC;  $v_s$  – the ship's speed.

Having determined the structure and amount of the ship general expenses, and distinguished the component that does not depend upon the ship's speed:

then, summarized it with the costs that depend upon the ship's speed:

$$\$_2(v_s) = k \cdot v_s^m, \tag{13}$$

where k – the coefficient of proportionality; m – power index; we get a formula for a general dependence of the rate of the total operational expenses costs upon the ship's speed:

$$\$(v_s) = \$_1 + \$_2(v_s) = \$_1 + k \cdot v_s^m.$$
(14)

**The problem solution.** The total summarized expenditures for the voyage, with respect to (11-14) as a function of the ship's speed will constitute:

$$C_{o}(v_{s}) = \$(v_{s}) \cdot t = \$_{1} \cdot t + \$_{2}(v_{s}) \cdot t = \$_{1} \cdot t + k \cdot v_{s}^{m} \cdot t, \qquad (15)$$

where t – the duration of the voyage,  $t(v_s) = \frac{D_{is}}{v_s}$ ;  $D_{is}$  – the distance or length of the voyage or a trip. Then:

$$C_{o}(v_{s}) = \$_{1} \cdot \frac{D_{is}}{v_{s}} + k \cdot v_{s}^{m} \cdot \frac{D_{is}}{v_{s}} = \$_{1} \cdot \frac{D_{is}}{v_{s}} + k \cdot v_{s}^{m-1} \cdot D_{is}, \qquad (16)$$

The voyage profit:

$$P_{r}(v_{s}) = I_{n}(v_{s}) - C_{o}(v_{s}), \qquad (17)$$

$$P_r(v_s) = m_0 \cdot c_0 \cdot e^{-n \cdot \frac{D_{is}}{v_s}} - \left(\$_1 \cdot \frac{D_{is}}{v_s} + k \cdot v_s^{m-1} \cdot D_{is}\right).$$
(18)

From where, obviously, the first component member will be having a limiting value in the view of a horizontal asymptote, shown on the fig. 1; the second component member is a hyperbolic one dependently upon the ship's speed; that is as the speed increases the first will grow up to the limiting value of the initial cost of the goods and loads  $C_0$ , and the second one will decrease down to zero. The third component member, in its turn, will increase the summarized total operational costs for the voyage, because of  $m \approx 3$ . Therefore, the summarized total operational costs for the voyage will have the extremum by the ship's speed in the view of minimum. Corresponding speed will be the optimal commercial one, if there are incomes constant in time; i.e. such incomes that do not depend upon the ship's speed. That speed may be chosen for the given voyage accordingly to the voyage task; although, it should be taken into consideration the restrictions for acceptable diapasons of parameters at the SPPPs operation. Firstly, the working point of the ME must lie in the boundaries of the field for the working parameters choice in accordance with the load diagram, as it shown in the «MAN Diesel» concern project guide for the ME.

In this problem setting, the optimal commercial speed will be the one that satisfies the maximum of the profit since the incomes increase as the speed increases but to a certain extent or limit. Of course, the restrictions to the working point of the ME there should be the same as to the previous case, namely, the main one of the requirements is that the working point of the ME has to be positioned in the boundaries of the field for the working parameters choice in accordance with the load diagram.

Such theoretical speculations are illustrated by the graphs shown on the fig. 1.

The optimal commercial speed, when having financial-economical data for the ship's incomes and operational costs, may be found in a graphical, as it is shown on the fig. 1, and analytical way.

When applying the analytical way of the optimal commercial speed finding, there used the necessary condition of a function extremum existence. The first derivative of the profit function with respect to the ship's speed:

$$\frac{dP_r(v_s)}{dv_s} = m_0 \cdot c_0 \cdot e^{-n \cdot \frac{D_{is}}{v_s}} \cdot n \cdot \frac{D_{is}}{v_s^2} + \$_1 \cdot \frac{D_{is}}{v_s^2} - k \cdot (m-1) \cdot v_s^{m-2} \cdot D_{is}.$$
 (19)



From where it may be found the optimal commercial speed of  $v_{s \text{ opt}}$ .

Figure 1 – Dependences of profit, income, and total operational costs for a voyage upon the ship's speed and the optimal commercial speed

**Practical application of the problem solution, and the researches results.** In order to calculate the voyage incomes (9) for determination the profit of (1-3, 17, 18), using the procedure of composing and solving the differential equation of (4-10), let us use a given level of rent. At the given level of rent, having found the value magnitude of the operational expenses for a voyage at the nominal ship's speed, we get the income.

Thus, for the beginning one has to find the total summarized operational expenses. For that purpose, let us determine the minimum specific fuel-oil consumption. In order to determine the part member (13) of the expressions of (14, 15) the SBFOC for the SSE has to be found. Applying the concepts of the «MAN Diesel» concern project guide for the SBFOC of the ME described in [1], we find the dependence of the SBFOC upon the output for the ME optimised at a point of O, lying upon a certain propeller curve, by the corresponding nomogram. Then, we get the hourly fuel-oil consumption. Transforming it, using prices for the fuel-oils and interrelationships between the ME outputs and the ship's speeds, into the dependences of the hourly fuel-oil consumptions upon the ship's speeds, we get that kind of the expenses as the function of the ship's speeds. Moreover, through the fuel-oil prices we may take into account all total summarized expenditures related to the ship's motion.

For example, considering a transport refrigerator equipped with the ME of 6L42MC, we have got for the voyage distance of up to 10,000 nautical miles  $v_{sC_{a}} = 17.36 \dots 17.37$  knots,  $v_{sP_{r}} = 17.9$  knots. Accordingly with the field for the working parameters choice, the approximate diapason for the ME power output, when working by the nominal propeller curve, constitute:  $N_e \approx 3,000 \dots 5,970$  kW. At this the SSE output, the corresponding ship's speed:  $v_s \approx 14 \dots 17.3$  knots.

In addition, the influence of subjective preferences factors could be made allowance for by the use of the formula of the complete probability where hypotheses are considered preferences to accept the corresponding policy of the operation.

The responsible manager has to have got reliable information about the strategy situations in the corresponding operational conditions. If the set of possible alternatives forms a complete group of non-conjunctive events, the complete probability with respect to the subjective preferences:

$$P(A) = \sum_{i=1}^{n} \pi_i P(A|\pi_i), \qquad (20)$$

where A – the preferred alternative concerning the ship's speed;  $\pi_i$  – the subjective preferences of the possible alternatives;  $P(A|\pi_i)$  – the conditional probability of the preferred alternative realization on the basis of that the corresponding operational mode has been preferred.

Then the expectation of a voyage profit would be:

$$Exp[P_r] = \sum_{i=1}^{n} P_{r_i} p_i, \qquad (21)$$

where  $P_{r_i}$  – the profit corresponding to the alternative;  $p_i$  – the complete probability of the corresponding alternative in accordance with the formula (20).

In the case of a discrete set of two possible variants we have the two alternatives: A – to except or B – to reject the possible optimal speed. Let it means the event of A:  $v_{s \text{ opt}} = 17.9$  knots or B:  $v_s = 17.3$  knots.

Accordingly to the given data and the subjective preferences of:  $\pi_A = 0.8$ ;  $\pi_B = 0.2$ ; and conditional probabilities of:  $P(A|\pi_A) = 0.5$ ;  $P(B|\pi_A) = 0.5$ ;  $P(A|\pi_B) = 0.15$ ;  $P(B|\pi_B) = 0.85$ ; events: A: the profit would constitute up to 432,024 \$ U.S.; B: 430,423 \$ U.S. per a voyage of 10,000 nautical miles; the expectation referred to the formula (21) yields the result of 431,111 \$ U.S. per the voyage.

**Conclusions and prospects of further researches.** Accordingly to the given simplified problem setting (1-19) and subjective preferences aspects (20, 21), it is considered the economical modes of operation for the SPPPs equipped with the ME of 6L42MC with respect to the subjective preferences. The optimal commercial speed, found on the condition of the minimum operational

expenses at the constant incomes, lies practically on the boundary of the limited diapason stipulated by the brake mean effective pressure for a stable and reliable work of the ME of 6L42MC. The optimal commercial speed, found on the condition of the maximum of the profit, exceeds the nominal one by 0.6 knots, therefore the ME is technically and economically substantiated to be operated at the maximum continuous rating of 5.970 kW for the ME output and the ship's speed of  $v_s = 17.3$  knots. The profit would constitute up to 430,423 \$ U.S. per a voyage of 10,000 nautical miles.

The further researches allow getting valuable information needed for better logistics. In the prospect they would lead to a proper choice of the operational modes for ship propulsion accordingly to the type of the vessel's power plant, properties of the transported load, and financial operational factors.

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### Гончаренко А.В. ОПТИМАЛЬНА КОМЕРЦІЙНА ШВИДКІСТЬ ТРАНСПОРТНОГО СУДНА З УРАХУВАННЯМ СУБ'ЄКТИВНИХ ПЕРЕВАГ ЕКСПЛУАТАЦІЙНИКІВ

Відповідно до технічних даних на судно та його двигун розглянуто економічні режими експлуатації суднової енергетичної установки обладнаної головним двигуном *6L42MC*. Проведено математичне моделювання оптимальної комерційної швидкості судна. В якості критеріїв оптимізації обрано мінімальні експлуатаційні витрати та максимальний прибуток за рейс. До урахування приймалися питома ефективна витрата палива, суб'єктивні переваги експлуатаційників. Отримано аналітичні параметрів, вирази що використовуються при оптимізації. Залежності проілюстровані графіками.

Ключові слова: суднова пропульсивна установка, головний двигун, режим експлуатації, оптимальна комерційна швидкість, суб'єктивні переваги, багатоальтернативні ситуації, математичне очікування.

#### **Гончаренко А.В.** ОПТИМАЛЬНАЯ КОММЕРЧЕСКАЯ СКОРОСТЬ ТРАНСПОРТНОГО СУДНА С УЧЕТОМ СУБЪЕКТИВНЫХ ПРЕДПОЧТЕНИЙ ЭКСПЛУАТАЦИОННИКОВ

Согласно техническим данным на судно и его двигатель рассмотрены экономичные эксплуатации судовой энергетической режимы установки оборудованной главным двигателем 6L42MC. Проведено математическое моделирование оптимальной коммерческой скорости судна. В качестве критериев оптимизации выбраны минимальные эксплуатационные расходы и максимальная прибыль за рейс. В учет принимались удельный эффективный расход топлива, субъективные предпочтения эксплуатационников. Получены аналитические параметров, используемых выражения при оптимизации. Зависимости проиллюстрированы графиками.

Ключевые слова: судовая пропульсивная установка, главный двигатель, режим эксплуатации, оптимальная коммерческая скорость, субъективные предпочтения, многоальтернативные ситуации, математическое ожидание.